

Cosmological solutions and observational constraints on 5-dimensional braneworld cosmology with gravitating Nambu-Goto matching conditions

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ABSTRACT: We investigate the cosmological implications of the recently constructed 5-dimensional braneworld cosmology with gravitating Nambu-Goto matching conditions. Inserting both matter and radiation sectors, we first extract the analytical cosmological solutions. Additionally, we use observational data from Type Ia Supernovae (SNIa) and Baryon Acoustic Oscillations (BAO), along with requirements of Big Bang Nucleosynthesis (BBN), in order to impose constraints on the parameters of the model. We find that the scenario at hand is in very good agreement with observations, and thus a small departure from the standard Randall-Sundrum scenario is allowed.

KEYWORDS: Braneworld models, dark energy, observational constraints, inflation

Contents

1	Introduction	2
2	5-dimensional braneworld with gravitating Nambu-Goto matching conditions	3
3	Cosmological solutions	5
4	Observational constraints	8
4.1	Branch $\epsilon = -1$	8
4.2	Branch $\epsilon = +1$	14
5	Conclusions	16
A	Observational data and constraints	17

1 Introduction

The standard approach for obtaining the equations of motion of a classical codimension-1 defect which backreacts on the bulk dynamics is well-known. It consists in considering the Einstein term or some gravity modification on the left-hand side of the field equation and all the matter content with the localized brane energy-momentum included on the right-hand side. Extracting the distributional pieces of this equation gives the Israel matching conditions [1]. Equivalently, one takes as usually the variation of the bulk action with respect to the bulk metric in order to get the bulk equations of motion and the variation of the brane-bulk action with respect to the bulk metric at the brane position (which coincides with the induced metric variation in the adapted frame) in order to get the junction conditions. Unfortunately, an analogous treatment for a generic distributional stress-energy tensor supported on a higher codimension defect leads to known inconsistencies for Einstein's equations [2–4] (a pure brane tension is a special situation which is consistent [5–13]). An idea to avoid this inconsistency is that the defect construction is not problematic, but rather Einstein theory does not have the necessary differential complexity to describe complicated distributional solutions. In [14] consideration of the general second derivative gravity theory in six dimensions (Einstein-Gauss-Bonnet) gave geometric matching conditions for a codimension-2 conical defect fueled by a generic energy-momentum tensor. The whole system of bulk field equations plus matching conditions was shown in [15] to be consistent for an axially symmetric codimension-2 cosmological brane.

A criticism to the standard treatment for obtaining the equations of motion of a defect was performed in [16]. The arguments were basically the need or the desire for consistency of the various codimension distributional defects and the existence of a meaningful equation of motion at the probe limit, things that seem to lack from the standard approach. Concerning the first point of consistency, the spirit of the proposal mentioned above is to include higher Lovelock densities [17, 18] to accommodate higher codimension defects, but for example, there is no higher than the second Lovelock density in six dimensions. Therefore, it is quite probable that in a D -dimensional spacetime the inclusion of the maximal $[(D-1)/2]$ Lovelock density (possibly along with lower Lovelock densities) makes branes with codimensions $\delta = 1, 2, \dots, [(D-1)/2]$ consistent, while for even higher codimensions the consistency seems questionable. Additionally, four-dimensions which represent effectively spacetime at certain length and energy scales are known not to allow generic codimension-2 or 3 defects. Concerning the second point of the probe limit equation of motion, it is obvious that the conventional matching conditions do not accept the Nambu-Goto probe limit. Since a test brane moving in a curved background spacetime traces a minimal surface in the lowest order approximation, it is natural to expect that when the self-gravitational field of the brane starts to be taken into account, a small correction should result on top of the minimal surface motion of the test approximation. Not only this, but even the geodesic limit of the Israel matching conditions is not an acceptable probe limit since being the geodesic equation a kinematical fact it should be preserved independent of the gravitational theory (similarly to [19], [20]) or the codimension of the defect, which is not the case for these matching conditions [14, 21–25]. Moreover, even the non-geodesic

probe limit of the standard equations of motion for various codimension defects in Lovelock gravity theories is not accepted, since this consists of higher order algebraic equations in the extrinsic curvature, and therefore, a multiplicity of probe solutions arise instead of a unique equation of motion at the probe level.

In view of the above difficulties it was suggested in [16] that maybe alternative matching conditions should be considered. These were proposed to be the “gravitating Nambu-Goto matching conditions” which arise by varying the brane-bulk action with respect to the brane embedding fields in a way that takes into account the gravitational back-reaction of the brane to the bulk. These alternative conditions may be close to the correct direction of finding realistic matching conditions since they seem to always satisfy the requirements of consistency and Nambu-Goto probe limit (the codimension-2 case was studied in [16], [26], while the codimension-1 in [27]). In [27] the application of the alternative matching conditions led to a new 5-dimensional braneworld cosmology which generalizes the conventional braneworld cosmology [28] in the sense that it contains an extra integration constant, and vanishing this constant gives back the standard braneworld cosmology.

In the current work we try to confront this cosmology with the current cosmological observational data (SNIa, BAO, BBN) in order to construct the corresponding probability contour-plots for the parameters of the theory. The paper is organized as follows: In section 2 we briefly present the alternative matching conditions and the basic features behind these. In section 3 we find in the cosmological framework the equation for the expansion rate including both the matter and radiation sectors. In section 4 we impose the observational constraints on the parameters of the model. Finally, a summary of the obtained results is given in section 5 of conclusions.

2 5-dimensional braneworld with gravitating Nambu-Goto matching conditions

Our system is described by five-dimensional Einstein gravity coupled to a localized 3-brane source. The domain wall Σ is assumed to be Z_2 -symmetric, it splits the spacetime \mathcal{M} into two parts \mathcal{M}_\pm and the two sides of Σ are denoted by Σ_\pm . The total brane-bulk action is

$$S = \int_{\mathcal{M}} d^5x \sqrt{|g|} (M^3 \mathcal{R} - \Lambda) - V \int_{\Sigma} d^4\chi \sqrt{|h|} - 2M^3 \int_{\Sigma_\pm} d^4\chi \sqrt{|h|} K + \int_{\Sigma} d^4\chi L_{mat}, \quad (2.1)$$

where $g_{\mu\nu}$ is the (continuous) bulk metric tensor and $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ is the induced metric on the brane with n^μ the unit normals pointing inwards \mathcal{M}_\pm (μ, ν, \dots are five-dimensional coordinate indices). The bulk coordinates are x^μ and the brane coordinates are χ^i (i, j, \dots are coordinate indices on the brane). The brane tension is $V > 0$ and the matter Lagrangian of the brane is L_{mat} . The only matter content of the bulk is the cosmological constant $\Lambda < 0$ and the higher dimensional mass scale is M . The contribution on each side of the wall of the Gibbons-Hawking term is also necessary here as in the standard derivation of the matching conditions. $K = h^{\mu\nu} K_{\mu\nu}$ is the trace of the extrinsic curvature $K_{\mu\nu} = h^\kappa_\mu h^\lambda_\nu n_{\kappa;\lambda}$ (the covariant differentiation ; corresponds to $g_{\mu\nu}$).

Varying (2.1) with respect to the bulk metric we get the bulk equations of motion

$$\mathcal{G}_{\mu\nu} = -\frac{\Lambda}{2M^3}g_{\mu\nu}, \quad (2.2)$$

where $\mathcal{G}_{\mu\nu}$ is the bulk Einstein tensor. In this variation, beyond the basic terms proportional to $\delta g_{\mu\nu}$ which give (2.2), there appear, as usually, extra terms proportional to the second covariant derivatives $(\delta g_{\mu\nu})_{;\kappa\lambda}$ which lead to a surface integral on the brane with terms proportional to $(\delta g_{\mu\nu})_{;\kappa}$. Adding the Gibbons-Hawking term, the normal derivatives of $\delta g_{\mu\nu}$, i.e. terms of the form $n^\kappa(\delta g_{\mu\nu})_{;\kappa}$, are canceled, and considering as boundary condition for the variation of the bulk metric its vanishing on the brane (Dirichlet boundary condition for $\delta g_{\mu\nu}$) there is nothing left beyond the terms in equation (2.2). The Gibbons-Hawking term will again contribute in the following variation performed in order to obtain the brane equations of motion.

According to the standard method, the interaction of the brane with the bulk comes from the variation $\delta g_{\mu\nu}$ at the brane position of the action (2.1), which is equivalent to adding on the right-hand side of equation (2.2) the term $\kappa_5^2 \tilde{T}_{\mu\nu} \delta^{(1)}$, where $\tilde{T}_{\mu\nu} = \sqrt{|h|/|g|} (T_{\mu\nu} - \lambda h_{\mu\nu})$, $T_{\mu\nu}$ is the brane energy-momentum tensor and $\delta^{(1)}$ is the one-dimensional delta function with support on the defect. This approach leads to the standard Israel matching conditions. Here, we discuss an alternative approach where the interaction of the brane with bulk gravity is obtained by varying the total action (2.1) with respect to δx^μ , the embedding fields of the brane position [16]. The embedding fields are some functions $x^\mu(\chi^i)$ and their variations are $\delta x^\mu(x^\nu)$. While in the standard method the variation of the bulk metric at the brane position remains arbitrary, here the corresponding variation is induced by δx^μ

$$\delta g_{\mu\nu} = \delta x g_{\mu\nu} = g'_{\mu\nu}(x^\rho) - g_{\mu\nu}(x^\rho) = -(g_{\mu\nu,\lambda} \delta x^\lambda + g_{\mu\lambda} \delta x^\lambda_{,\nu} + g_{\nu\lambda} \delta x^\lambda_{,\mu}) = -\mathcal{L}_{\delta x} g_{\mu\nu}, \quad (2.3)$$

and is obviously independent from the variation leading to (2.2). The result of this variation gives the codimension-1 gravitating Nambu-Goto matching conditions [27] (for a reminiscent variation see also [29])

$$\left[K^{ij} - K h^{ij} + \frac{1}{4M^3} (T^{ij} - V h^{ij}) \right] K_{ij} = 0 \quad (2.4)$$

$$T^{ij}_{|j} = -4M^3 (K^{ij} - K h^{ij})_{|j}, \quad (2.5)$$

where $K_{ij} = K_{ij}^+ = K_{ij}^-$, $K^{\mu\nu} = K^{ij} x^\mu_{,i} x^\nu_{,j}$ and $|$ denotes covariant differentiation with respect to $h_{\mu\nu}$. These equations are supplemented with the bulk equations which are defined limitingly on the brane, and therefore, additional equations have to be satisfied at the brane position beyond the matching conditions. Using these bulk equations the system of the above matching conditions (2.4), (2.5) is written equivalently as

$$(T^{ij} - V h^{ij}) K_{ij} = 4(M^3 R - \Lambda) \quad (2.6)$$

$$T^{ij}_{|j} = 0, \quad (2.7)$$

where R is the 3-dimensional Ricci scalar.

3 Cosmological solutions

In order to search for cosmological solutions we consider the corresponding form for the bulk metric in the Gaussian-normal coordinates

$$ds_5^2 = dy^2 - n^2(t, y)dt^2 + a^2(t, y) \gamma_{\hat{i}\hat{j}}(\chi^{\hat{\ell}}) d\chi^{\hat{i}} d\chi^{\hat{j}}, \quad (3.1)$$

where $\gamma_{\hat{i}\hat{j}}$ is a maximally symmetric 3-dimensional metric ($\hat{i}, \hat{j}, \dots = 1, 2, 3$) characterized by its spatial curvature $k = -1, 0, 1$. The energy-momentum tensor on the brane T_{ij} (beyond that of the brane tension V) is assumed to be the one of perfect cosmic fluids with total energy density ρ and total pressure p .

The ty , yy bulk equations (2.2) at the position of the brane are

$$\dot{A} + nH(A - N) = 0 \quad (3.2)$$

$$A(A + N) - (X + Y) + \frac{\Lambda}{6M^3} = 0, \quad (3.3)$$

where

$$\begin{aligned} A &= \frac{a'}{a}, \quad N = \frac{n'}{n}, \\ H &= \frac{\dot{a}}{na}, \\ X &= H^2 + \frac{k}{a^2}, \\ Y &= \frac{\dot{H}}{n} + H^2 = \frac{\dot{X}}{2nH} + X, \end{aligned} \quad (3.4)$$

and a prime, a dot denote respectively $\partial/\partial y$, $\partial/\partial t$. The cosmic scale factor, lapse function and Hubble parameter arise as the restrictions on the brane of the functions $a(t, y)$, $n(t, y)$ and $H(t, y)$ respectively. Other quantities also have their corresponding values when restricted on the brane, and since all the following equations will refer to the brane position, we will use the same symbols for the restricted quantities without confusion. Eliminating N between equations (3.2), (3.3) we get

$$\left(A^2 a^4 - X a^4 + \frac{\Lambda}{12M^3} a^4 \right)' = 0. \quad (3.5)$$

The integration of (3.5) gives the equation

$$A^2 - X + \frac{\Lambda}{12M^3} + \frac{\mathcal{C}}{a^4} = 0 \quad (3.6)$$

(\mathcal{C} is integration constant), with two branches for A

$$A = \pm \sqrt{X - \frac{\mathcal{C}}{a^4} - \frac{\Lambda}{12M^3}}. \quad (3.7)$$

The matching condition (2.6) becomes

$$3(p - V)A - (\rho + V)N = 24M^3(X + Y) - 4\Lambda, \quad (3.8)$$

from which the quantity N could also be found. Combining equations (3.3), (3.8) to eliminate N , we obtain the following algebraic equation for A

$$(\rho + 3p - 2V)A^2 - 4[6M^3(X + Y) - \Lambda]A - (\rho + V)\left(X + Y - \frac{\Lambda}{6M^3}\right) = 0. \quad (3.9)$$

Substituting A from (3.7) in (3.9), we obtain the final Raychaudhuri equation for the brane cosmology

$$\frac{\dot{H}}{n} + 2H^2 + \frac{k}{a^2} - \frac{\Lambda}{6M^3} = \frac{\rho + 3p - 2V}{4M^3} \frac{H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} - \frac{\Lambda}{12M^3}}{\frac{\rho+V}{4M^3} \pm 6\sqrt{H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} - \frac{\Lambda}{12M^3}}}. \quad (3.10)$$

It is seen from (3.10) that for $\mathcal{C} = k = \rho = p = 0$, the lower branch contains the Minkowski solution under the assumption of the Randall-Sundrum fine-tuning $\Lambda + V^2/(12M^3) = 0$ [30, 31]. We will not assume this condition in our analysis, so in the absence of matter our cosmology may have a de-Sitter vacuum. It is assumed that the quantity inside the square root of equation (3.10) is positive.

In [27] a single component perfect fluid was considered. Here, since we want to confront the model with real data, we will be more precise by assuming that the total energy density ρ consists of the matter component ρ_m with $p_m = 0$ and the radiation component ρ_r with $p_r = \frac{1}{3}\rho_r$, i.e. $\rho = \rho_m + \rho_r$. Now, the integration process of (3.10) differs from that in [27]. The variable

$$\Xi = \frac{1}{2} \ln \left[\frac{12M^3}{-\Lambda} \left(H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} - \frac{\Lambda}{12M^3} \right) \right] \quad (3.11)$$

obeys the differential equation

$$\frac{d\Xi}{d \ln a} = \frac{\tilde{\rho} + 3\tilde{p}}{\tilde{\rho} \pm 6e^\Xi} - 2, \quad (3.12)$$

where

$$\tilde{\rho} = \sqrt{\frac{12M^3}{-\Lambda}} \frac{\rho + V}{4M^3} = \frac{\rho}{\rho_*} + \tilde{V} \quad (3.13)$$

$$\tilde{p} = \sqrt{\frac{12M^3}{-\Lambda}} \frac{p - V}{4M^3} = \frac{p}{\rho_*} - \tilde{V} \quad (3.14)$$

$$\tilde{V} = \frac{V}{\rho_*} \quad (3.15)$$

$$\rho_* = 4M^3 \sqrt{\frac{-\Lambda}{12M^3}}. \quad (3.16)$$

Note that the Randall-Sundrum fine-tuning corresponds to the value $\tilde{V} = 3$. Using the conservation equation (2.7) in the standard form

$$\dot{\rho} + 3nH(\rho + p) = 0, \quad (3.17)$$

we obtain the equation

$$\frac{d\tilde{\rho}}{d \ln a} + 3(\tilde{\rho} + \tilde{p}) = 0. \quad (3.18)$$

Finally, changing to the variable

$$\Phi = (\tilde{\rho} \pm 6e^\Xi)^2, \quad (3.19)$$

we get from (3.12), (3.18), after some cancelations, the differential equation

$$\frac{d\Phi}{d \ln a} + 4\Phi = -2\tilde{\rho}(\tilde{\rho} + 3\tilde{p}). \quad (3.20)$$

Each fluid component is conserved independently

$$\dot{\rho}_m + 3nH(\rho_m + p_m) = 0 \quad , \quad \dot{\rho}_r + 3nH(\rho_r + p_r) = 0, \quad (3.21)$$

so the solutions are

$$\rho_m = \frac{\rho_{m0}}{a^3} \quad , \quad \rho_r = \frac{\rho_{r0}}{a^4}. \quad (3.22)$$

Therefore, equation (3.20) becomes a linear differential equation in terms of a

$$\frac{d\Phi}{d \ln a} + 4\Phi = -\frac{2}{\rho_*^2} \left(\frac{\rho_{m0}}{a^3} + \frac{\rho_{r0}}{a^4} + V \right) \left(\frac{\rho_{m0}}{a^3} + 2\frac{\rho_{r0}}{a^4} - 2V \right), \quad (3.23)$$

with general solution

$$\Phi = \frac{1}{\rho_*^2} [(\rho_m + \rho_r + V)^2 - 2V\rho_r] + \frac{\tilde{c}}{a^4}, \quad (3.24)$$

where \tilde{c} is integration constant.

From the definition (3.19) we can find that

$$\tilde{\rho} \pm \frac{24M^3}{\rho_*} \sqrt{H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} - \frac{\Lambda}{12M^3}} = \epsilon \sqrt{\Phi}. \quad (3.25)$$

In this equation the sign index $\epsilon = +1$ or -1 has been used to denote a new different bifurcation from the previous \pm branches. It is seen from (3.25) that the sign $\epsilon = -1$ is only consistent with the lower \pm branch, while the sign $\epsilon = +1$ is consistent with both \pm branches. The distinction, however, introduced by the sign index \pm will be lost in the expressions for the expansion rate and the acceleration parameter and only the sign ϵ will distinguish the two branches of solutions.

The *expansion rate* of the new cosmology arises by squaring equation (3.25) and is given by

$$H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \left(\frac{\rho_*}{24M^3} \right)^2 \left\{ \left[\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} - \epsilon \sqrt{\left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right)^2 - 2\tilde{V}\frac{\rho_r}{\rho_*} + \frac{\tilde{c}}{a^4}} \right]^2 - 36 \right\}, \quad (3.26)$$

where in (3.26) one can set $\rho_r = 0$. Redefining the integration constant \tilde{c} as $\mathbf{c} = \frac{\rho_*}{\rho_{r0}} \tilde{c} - 2\tilde{V}$, the expansion rate can also be written as

$$H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \left(\frac{\rho_*}{24M^3} \right)^2 \left\{ \left[\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} - \epsilon \sqrt{\left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right)^2 + \mathbf{c}\frac{\rho_r}{\rho_*}} \right]^2 - 36 \right\}, \quad (3.27)$$

where in (3.27) one cannot set $\rho_r = 0$ since ρ_{r0} is in the denominator of the definition of \mathfrak{c} . This solution contains two integration constants. The first constant \mathcal{C} is associated with the usual dark radiation term reflecting the non-vanishing bulk Weyl tensor. The second constant \tilde{c} or \mathfrak{c} is the new feature that does not appear in the cosmology of the standard matching conditions [28] and signals new characteristics in the cosmic evolution. Setting $\mathfrak{c} = 0 \Leftrightarrow \tilde{c} = \frac{2\tilde{V}\rho_{r0}}{\rho_*}$ in the branch $\epsilon = -1$ we obtain the braneworld cosmology of the standard matching conditions $H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \left(\frac{\rho_m + \rho_r + V}{12M^3}\right)^2 + \frac{\Lambda}{12M^3}$ (if there is no radiation we just set $\tilde{c} = 0$). Of course, there are always the extra integration constants ρ_{m0} , ρ_{r0} of equations (3.22) which are adjusted by the today matter contents, while the today Hubble value H_0 is assumed to be given. The solution also contains three free parameters M , V , Λ or M , \tilde{V} , ρ_* . In [27] for a single dust perfect fluid, which approximates well at least the late-times behaviour, it was found analytically for values of \tilde{V} extremely close to the Randall-Sundrum fine-tuning the position of the recent passage from a long deceleration era to the present accelerating epoch. Moreover, the age of the universe was estimated and the time variability of the dark energy equation of state was calculated.

4 Observational constraints

As we analyzed in detail above, the cosmological scenario at hand leads to the Friedmann equation (3.26), where the index $\epsilon = \pm 1$ corresponds to two branches of solutions. The Friedmann equation contains the following parameters: \mathcal{C} , \tilde{c} , M , \tilde{V} and ρ_* , along with Ω_{m0} , Ω_{r0} , Ω_{k0} . \mathcal{C} and \tilde{c} are integration constants, M is the fundamental 5D Planck mass, and the other two \tilde{V} , ρ_* are connected to the fundamental model parameters M , V and Λ through the relations (3.15), (3.16). The identification of Newton's constant G_N in equation (3.26) as a combination of the model parameters will reduce the number of these parameters by one. Then, using G_N we will define the various density parameters.

4.1 Branch $\epsilon = -1$

The scale factor for the branch $\epsilon = -1$ with $\tilde{V} < 3$ is bounded from above and we will not consider this case in detail. However, the branch $\epsilon = -1$ with $\tilde{V} \geq 3$ possesses the late-times asymptotic linearized regime (that is when $\rho_m + \rho_r \ll \rho_* \tilde{V}$, $\rho_r/\rho_{r0} \ll \tilde{V}^2/\tilde{c}$) with a positive effective cosmological constant

$$H^2 + \frac{k}{a^2} \approx \frac{\Lambda_{eff}}{3} + 2\gamma\rho_m + \gamma\rho_r + \left(\mathcal{C} + \frac{\gamma\rho_*\tilde{c}}{2\tilde{V}}\right)\frac{1}{a^4}, \quad (4.1)$$

where

$$\gamma = \frac{V}{144M^6} \quad (4.2)$$

$$\Lambda_{eff} = 3\left(\frac{\rho_*}{4M^3}\right)^2 \left(\frac{\tilde{V}^2}{9} - 1\right) = \frac{1}{4M^3}\left(\Lambda + \frac{V^2}{12M^3}\right). \quad (4.3)$$

Now, as usual in braneworld or other modified gravity models, from this late-times Friedmann equation, one reads the Newton's constant. Since asymptotically the coefficients of

ρ_m, ρ_r in (4.1) are different, and $\rho_r \ll \rho_m$, we associate Newton's constant with ρ_m

$$\gamma = \frac{V}{144M^6} \equiv \frac{4\pi G_N}{3}. \quad (4.4)$$

With this identification we can go back to the full Friedmann equation (3.26) and reduce one parameter, for instance M . Thus, the expansion rate (3.26) for $\epsilon = -1$, $\tilde{V} \geq 3$ becomes

$$H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \frac{\pi G_N \rho_*}{3\tilde{V}} \left\{ \left[\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} + \sqrt{\left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right)^2 - 2\tilde{V} \frac{\rho_r}{\rho_*} + \frac{\tilde{\mathcal{C}}}{a^4}} \right]^2 - 36 \right\}. \quad (4.5)$$

Finally, in order to complete the steps we rewrite (4.5) as

$$H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \frac{8\pi G_N}{3} (\rho_m + \rho_r + \rho_{DE}) \quad (4.6)$$

with

$$\rho_{DE} = \frac{\rho_*}{8\tilde{V}} \left\{ \left[\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} + \sqrt{\left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right)^2 - 2\tilde{V} \frac{\rho_r}{\rho_*} + \frac{\tilde{\mathcal{C}}}{a^4}} \right]^2 - 36 \right\} - (\rho_m + \rho_r). \quad (4.7)$$

Note that this ρ_{DE} at late-times goes to $\frac{\Lambda_{eff}}{8\pi G_N} - \frac{\rho_r}{2} + \frac{\rho_* \tilde{\mathcal{C}}}{4\tilde{V} a^4}$ which asymptotically goes to $\frac{\Lambda_{eff}}{8\pi G_N}$, i.e. to a simple cosmological constant.

So now, we can define the various density parameters straightforwardly as

$$\Omega_m = \frac{8\pi G_N \rho_m}{3H^2} \quad (4.8)$$

$$\Omega_r = \frac{8\pi G_N \rho_r}{3H^2} \quad (4.9)$$

$$\Omega_{DE} = \frac{8\pi G_N \rho_{DE}}{3H^2} \quad (4.10)$$

$$\Omega_k = -\frac{k}{a^2 H^2} \quad (4.11)$$

$$\Omega_{\mathcal{C}} = \frac{\mathcal{C}}{a^4 H^2}. \quad (4.12)$$

Finally, assuming that the present scale factor is $a_0 = 1$ and using the redshift as the independent variable ($1/a = 1+z$), we can write the Friedmann equation (4.6) in the usual form, convenient to observational fittings

$$H^2 = H_0^2 \left\{ \Omega_{k0}(1+z)^2 + \Omega_{\mathcal{C}0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \frac{8\pi G_N \rho_{DE}(z)}{3H_0^2} \right\}. \quad (4.13)$$

Here, ρ_{DE} , according to (4.7), is

$$\begin{aligned} \rho_{DE}(z) = \frac{\rho_*}{8\tilde{V}} \left\{ \left[\frac{3H_0^2 \Omega_{m0}}{8\pi G_N \rho_*} (1+z)^3 + \frac{3H_0^2 \Omega_{r0}}{8\pi G_N \rho_*} (1+z)^4 + \tilde{V} + \mathcal{A}(z) \right]^2 - 36 \right\} \\ - \frac{3H_0^2 \Omega_{m0}}{8\pi G_N} (1+z)^3 - \frac{3H_0^2 \Omega_{r0}}{8\pi G_N} (1+z)^4, \end{aligned} \quad (4.14)$$

with

$$\mathcal{A}(z) = \sqrt{\left(\frac{3H_0^2\Omega_{m0}}{8\pi G_N\rho_*}(1+z)^3 + \frac{3H_0^2\Omega_{r0}}{8\pi G_N\rho_*}(1+z)^4 + \tilde{V}\right)^2 - \frac{3H_0^2\Omega_{r0}\tilde{V}}{4\pi G_N\rho_*}(1+z)^4 + \tilde{c}(1+z)^4}. \quad (4.15)$$

Alternatively, one could write the last term inside the curly bracket of (4.13) as $\Omega_{DE0}(1+z)^{3(1+w_{DE}(z))}$, with $\Omega_{DE0} = 1 - \Omega_{m0} - \Omega_{r0} - \Omega_{C0} - \Omega_{k0}$ and $w_{DE}(z)$ extracted from (4.14). This normalization at the current values fixes one of the parameters, e.g. Ω_{r0} .

In summary, Eq. (4.13) is the one we will fit, with \mathcal{C} , \tilde{c} , \tilde{V} , ρ_* and Ω_{m0} as parameters (for simplicity we fix H_0 and Ω_{k0} to their Planck + WP + highL + BAO best fit values, namely $\Omega_{k0} = -0.0003$ and $H_0 = 67.77\text{km s}^{-1}\text{Mpc}^{-1}$ [32]).

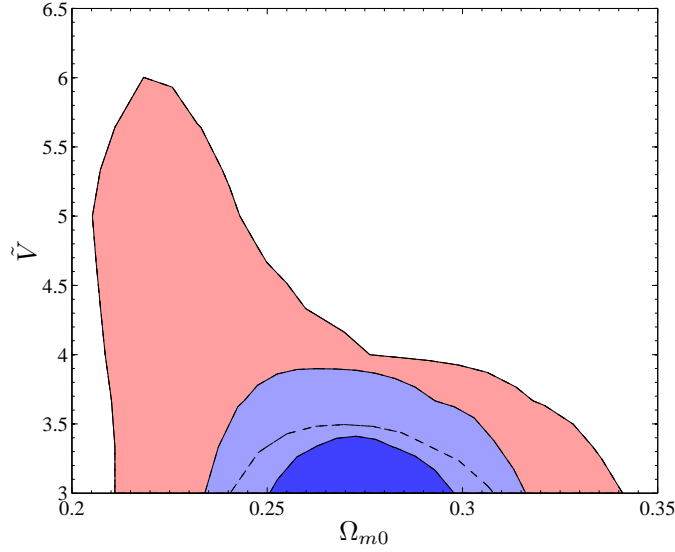


Figure 1. (Color Online) *Two-dimensional likelihood contours in the (Ω_{m0}, \tilde{V}) plane for the $\epsilon = -1$ branch and fixed \tilde{c} to its Randall-Sundrum value ($\tilde{c} = 2\tilde{V}\rho_{r0}/\rho_*$) from the SnIa (red and pink) and SnIa+BAO (blue and light blue) data combinations. The light regions (pink and light blue respectively) correspond to 2σ confidence level, while the darker regions (red and blue respectively) correspond to 1σ confidence level. Note that in this specific plot the 1σ bound of the SnIa (red) data combinations is inside the 2σ bound of the SnIa+BAO (light blue) data combinations.*

The \mathcal{C} -term in (4.6) corresponds to dark radiation, so it is proportional to $1/a^4$. This term, in particular Ω_{C0} , cannot be constrained efficiently by the low-redshift observations we are going to use in our analysis. However, since this dark radiation component was present at the time of Big Bang Nucleosynthesis (BBN) too, that is at redshift $z_{BBN} \sim 10^9$, we can use BBN arguments in order to constrain it. Specifically, the data impose an upper bound on the amount of total radiation (standard and exotic), which is expressed through the parameter ΔN_ν of the effective neutrino species [33–35]. Thus, in our case, this bound imposes a constraint on Ω_{C0} , namely

$$\Omega_{C0} = 0.135\Delta N_\nu\Omega_{r0}. \quad (4.16)$$

The recently released Planck results impose a quite tight constraint on the effective number of neutrino species [32]: $N_{\text{eff}} = 3.30^{+0.54}_{-0.51}$ (95% C.L.) from the Planck+WP+highL+BAO data combination. Therefore, the 95% C.L. upper limit of ΔN_ν is $\Delta N_\nu < 0.776$. This leads to a very tight constraint on the dark radiation component of the scenario at hand, namely $\Omega_{\mathcal{C}0} < 5 \times 10^{-6}$ (95% C.L.). Thus, we can safely neglect this term in the remaining analysis and the remaining parameters to be fitted are \tilde{c} , \tilde{V} , ρ_* and Ω_{m0} .

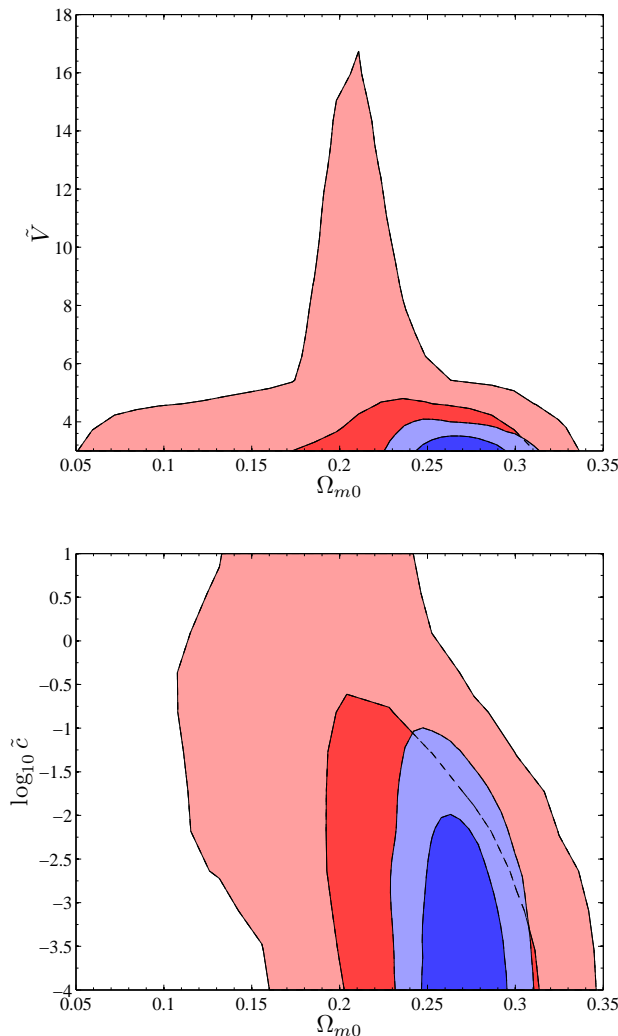


Figure 2. (Color Online) *Two-dimensional likelihood contours in the (Ω_{m0}, \tilde{V}) and $(\Omega_{m0}, \log_{10} \tilde{c})$ planes for the $\epsilon = -1$ branch from the SnIa (red and pink) and SnIa+BAO (blue and light blue) data combinations. The light regions (pink and light blue respectively) correspond to 2σ confidence level, while the darker regions (red and blue respectively) correspond to 1σ confidence level.*

As a starting analysis, let us fit the case where \tilde{c} is set to its value that corresponds to the standard braneworld cosmological scenario [28], namely $\tilde{c} = 2\tilde{V}\rho_{r0}/\rho_*$ (which is exactly zero in the absence of radiation). Thus, in this case we have only three free

parameters, namely \tilde{V} , ρ_* and Ω_{m0} . In Fig. 1 we provide the two-dimensional contour plots on (Ω_{m0}, \tilde{V}) , using SnIa and SnIa+BAO data combinations. The details of the fitting procedure are presented in the Appendix. As we observe, when we use SnIa data only, the constraints on \tilde{V} are relatively weak, namely $3 < \tilde{V} < 5.5$ at the 95% confidence level. However, addition of the BAO data introduces an extra constraining power and the total constraint becomes tighter, namely $3 < \tilde{V} < 3.4$ (95% C.L.) from SnIa+BAO data. Finally, as we describe in the Appendix, the efficiency of the fitting is quantified by χ^2 , which for this case is $\chi^2 \approx 570$.

Let us now proceed to the general case, that is considering \tilde{c} as an additional free parameter. In the upper graph of Fig. 2 we present the contour plots of \tilde{V} versus Ω_{m0} , while in the lower graph of Fig. 2 we depict the contour plots of \tilde{c} versus Ω_{m0} . As we observe, the SnIa constraints on the parameter \tilde{V} are much weaker than those of Fig. 1, due to the additional fitting variable. In particular, the 95% C.L. bound is $3 < \tilde{V} < 15.3$ (additionally note that the parameter space $\Omega_{m0} < 0.2$ is now allowed by the SnIa data, exactly due to the presence of non-zero \tilde{c}). Concerning \tilde{c} the SnIa data leads also to the relatively weak constraint $\log_{10} \tilde{c} < 0.1$ (95% C.L.). However, for the combined SnIa with BAO data, the constraints become much tighter. At 95% confidence level they are $3 < \tilde{V} < 3.7$ and $\log_{10} \tilde{c} < -1.6$, while their best fit values are very close to 3 and 0 respectively. Finally, the corresponding χ^2 is $\chi^2 \approx 570$.

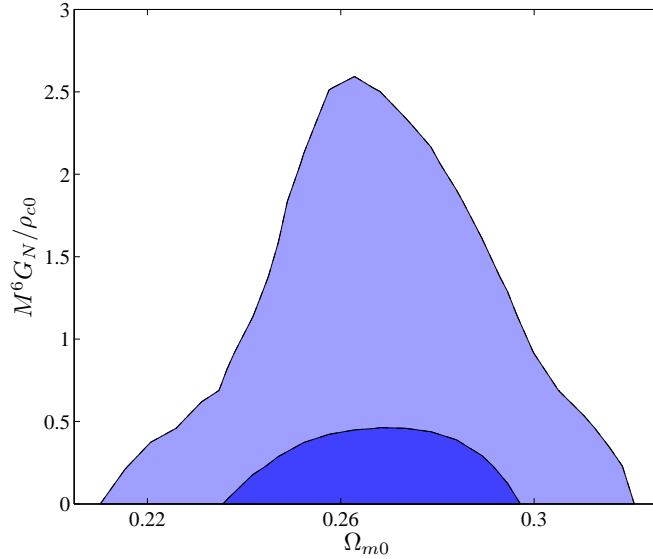


Figure 3. (Color Online) *Two-dimensional likelihood contours of the dimensionless quantity $M^6 G_N / \rho_{c0}$ versus Ω_{m0} , where ρ_{c0} is the current critical density, for the $\epsilon = -1$ branch from the SnIa+BAO data combinations. The lighter region corresponds to 2σ confidence level, while the darker region corresponds to 1σ confidence level.*

As we observe, the cosmological observations constrain \tilde{V} and \tilde{c} close to their Randall-Sundrum values, namely $\tilde{V} = 3$ and $\tilde{c} \approx 0$ ($\tilde{c} = 0$ in the case of radiation absence).

However, note that the data allow for a departure from Randall-Sundrum scenario. In particular, although the present model has an additional parameter compared to Randall-Sundrum one, the corresponding χ^2 is the same in two models. This means that braneworld models with gravitating Nambu-Goto matching condition are in a very good agreement with observations too.

Combining equations (3.15), (4.4) we obtain for the fundamental mass scale M the relation

$$M^6 = \frac{\tilde{V}\rho_*}{192\pi G_N}. \quad (4.17)$$

The likelihood contours of the dimensionless quantity $M^6 G_N / \rho_{c0}$ versus Ω_{m0} , where ρ_{c0} is the current critical density, is shown in Fig. 3. We can then straightforwardly estimate that at 1σ confidence level $0 < M < 0.042 \text{ GeV}$. Moreover, to give an estimate for the value of the brane tension V , we use the relation $V = 192\pi G_N M^6$, which leads to $0 < V < 2.22 \times 10^{-44} \text{ GeV}^4$ at 1σ confidence level. That is $0 < V < 0.87 \times 10^3 \rho_{\Lambda 0}$, where $\rho_{\Lambda 0}$ is the current value of the energy density of the observed cosmological constant.

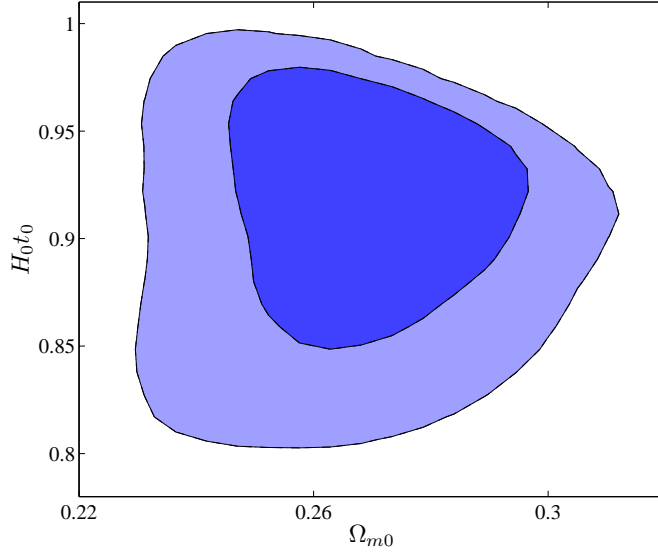


Figure 4. (Color Online) *Two-dimensional likelihood contours of $H_0 t_0$ versus Ω_{m0} for the $\epsilon = -1$ branch from the $\text{SnIa}+\text{BAO}$ data combinations. The lighter region corresponds to 2σ confidence level, while the darker region corresponds to 1σ confidence level.*

Finally, we close this subsection by examining the constraints on the model from the age of the universe. In general, the age of the universe is given by

$$t_0 = \int_0^\infty \frac{dz}{(1+z)H(z)}, \quad (4.18)$$

where in the scenario at hand $H(z)$ is given by equation (4.13). Thus, taking into account the constraints on the model parameters elaborated above, we can construct the contour

plots of $H_0 t_0$ versus Ω_{m0} , which is presented in Fig. 4. We can then straightforwardly estimate the age in Gyr, finding $12.23 \text{ Gyr} \leq t_0 \leq 14.13 \text{ Gyr}$ at 1σ confidence level (for the Λ CDM model with $\Omega_{m0} = 0.28$ the corresponding age is 13.5 Gyr). We observe from equations (3.26) and (4.18) that larger values of the mass scale M in the range found above correspond to larger values of the age of the universe. Thus, since larger ages are preferable, the most probable estimations for M lie closer to the upper bound.

4.2 Branch $\epsilon = +1$

In this case, the full Friedmann equation (3.26) is

$$H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \left(\frac{\rho_*}{24M^3} \right)^2 \left\{ \left[\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} - \sqrt{\left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right)^2 - 2\tilde{V} \frac{\rho_r}{\rho_*} + \frac{\tilde{c}}{a^4}} \right]^2 - 36 \right\}. \quad (4.19)$$

The branch $\epsilon = +1$ is completely new comparing to the standard braneworld models since the scale factor is bounded from above for any value of \tilde{V} . Therefore, contrary to the branch $\epsilon = -1$, here, there is no pure late-times linearization regime. However, expanding the expression (4.19), there is a term linear in ρ_m, ρ_r , so Newton's constant G_N can also here be identified. More precisely it is $H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \gamma(\rho_m + \frac{\rho_r}{2}) + \dots$, where \dots do not contain terms linear in ρ_m, ρ_r , and $\gamma = \frac{V}{144M^6}$. Therefore, associating G_N with ρ_m we have the identification

$$\gamma = \frac{V}{144M^6} \equiv \frac{8\pi G_N}{3}. \quad (4.20)$$

Going back to equation (4.19), we eliminate the parameter M and we rewrite the expansion rate for $\epsilon = +1$ as

$$H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \frac{4\pi G_N \rho_*}{3\tilde{V}} \left[\tilde{V} \frac{2\rho_m + \rho_r}{\rho_*} + \left(\frac{\rho_m + \rho_r}{\rho_*} \right)^2 + \tilde{V}^2 - 18 + \frac{\tilde{c}}{2a^4} - \left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right) \sqrt{\left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right)^2 - 2\tilde{V} \frac{\rho_r}{\rho_*} + \frac{\tilde{c}}{a^4}} \right]. \quad (4.21)$$

This expression takes the standard form

$$H^2 + \frac{k}{a^2} - \frac{\mathcal{C}}{a^4} = \frac{8\pi G_N}{3} (\rho_m + \rho_r + \rho_{DE}), \quad (4.22)$$

where

$$\rho_{DE} = \frac{\rho_*}{2\tilde{V}} \left[\left(\frac{\rho_m + \rho_r}{\rho_*} \right)^2 - \frac{\tilde{V} \rho_r}{\rho_*} + \tilde{V}^2 - 18 + \frac{\tilde{c}}{2a^4} - \left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right) \sqrt{\left(\frac{\rho_m + \rho_r}{\rho_*} + \tilde{V} \right)^2 - 2\tilde{V} \frac{\rho_r}{\rho_*} + \frac{\tilde{c}}{a^4}} \right]. \quad (4.23)$$

Defining the density parameters as in (4.8)-(4.12), we find equation (4.13), where $\rho_{DE}(z)$ is now given by

$$\rho_{DE}(z) = \frac{\rho_*}{2\tilde{V}} \left\{ \left(\frac{3H_0^2 \Omega_{m0}}{8\pi G_N \rho_*} (1+z)^3 + \frac{3H_0^2 \Omega_{r0}}{8\pi G_N \rho_*} (1+z)^4 \right)^2 - \frac{3H_0^2 \Omega_{r0} \tilde{V}}{8\pi G_N \rho_*} (1+z)^4 + \tilde{V}^2 - 18 + \frac{\tilde{c}}{2} (1+z)^4 - \left(\frac{3H_0^2 \Omega_{m0}}{8\pi G_N \rho_*} (1+z)^3 + \frac{3H_0^2 \Omega_{r0}}{8\pi G_N \rho_*} (1+z)^4 + \tilde{V} \right) \mathcal{A}(z) \right\}, \quad (4.24)$$

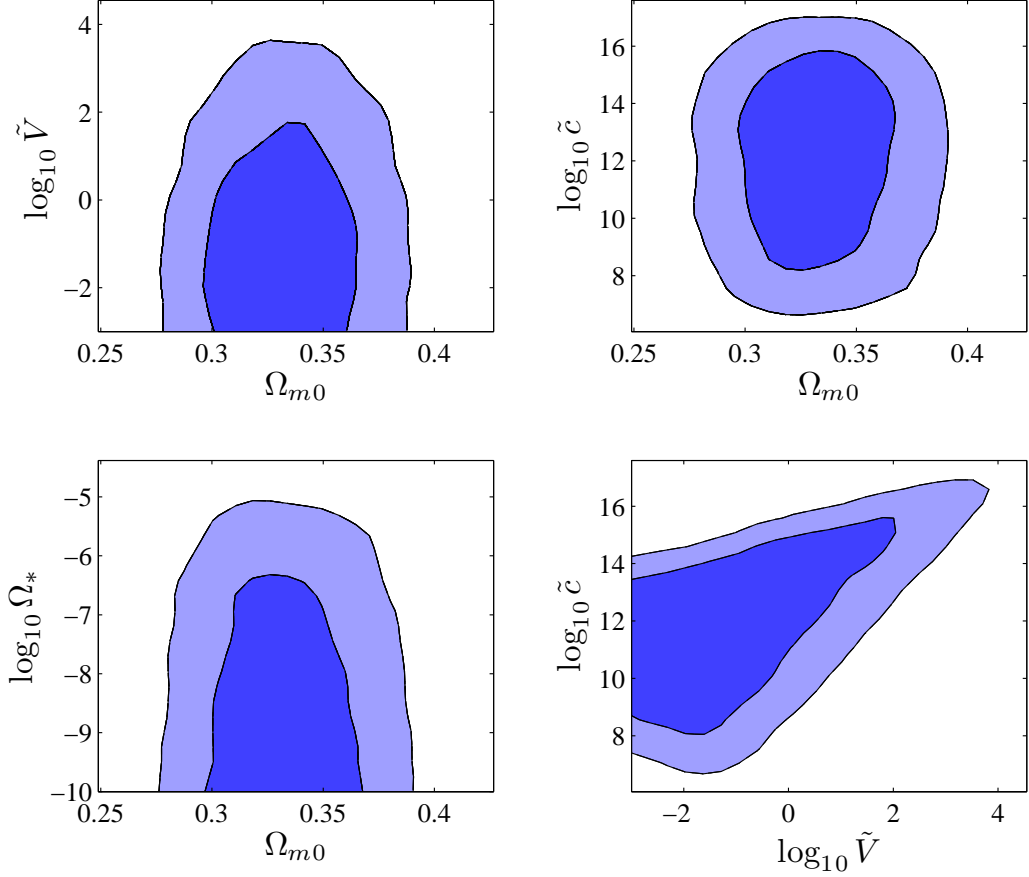


Figure 5. (Color Online) *Two-dimensional likelihood contours in the (Ω_{m0}, \tilde{V}) , $(\Omega_{m0}, \log_{10} \tilde{c})$, (Ω_{m0}, Ω_*) and $(\tilde{V}, \log_{10} \tilde{c})$ planes, for the $\epsilon = +1$ branch, from the *SnIa+BAO* data combinations. The lighter regions correspond to 2σ confidence level, while the darker region correspond to 1σ confidence level.*

with

$$\mathcal{A}(z) = \sqrt{\left(\frac{3H_0^2\Omega_{m0}}{8\pi G_N\rho_*}(1+z)^3 + \frac{3H_0^2\Omega_{r0}}{8\pi G_N\rho_*}(1+z)^4 + \tilde{V}\right)^2 - \frac{3H_0^2\Omega_{r0}\tilde{V}}{4\pi G_N\rho_*}(1+z)^4 + \tilde{c}(1+z)^4}. \quad (4.25)$$

In summary, Eq. (4.22) is the one we will fit, with \mathcal{C} , \tilde{c} , \tilde{V} , ρ_* and Ω_{m0} as parameters (again for simplicity we fix H_0 and Ω_{k0} to their (Planck+WP+highL+BAO) best fit values, namely $\Omega_{k0} = -0.0003$ and $H_0 = 67.77\text{km s}^{-1}\text{Mpc}^{-1}$ [32]). Similarly to the previous subsection, we can safely neglect \mathcal{C} since it is negligible according to BBN analysis. Finally, instead of ρ_* it proves more convenient to introduce the dimensionless quantity

$$\Omega_* \equiv \frac{\rho_*}{\rho_{c0}}, \quad (4.26)$$

where ρ_{c0} is the present critical energy density of the Universe.

We use combined SnIa and BAO data to constrain \tilde{c} , \tilde{V} , Ω_* and Ω_{m0} . In Fig. 5 we present the corresponding two-dimensional likelihood contours. Firstly, note that in this case \tilde{V} is not theoretically restricted to values greater than 3 and in particular it is constrained in much smaller values, namely $\log_{10} \tilde{V} < 2.0$ (95% C.L. upper limit). Additionally, note that since at late times ρ_{DE} acquires negative values, the constraint on Ω_* is very close to zero, namely $\log_{10} \Omega_* < -5.5$ (95% C.L.). Due to the strong degeneracy between Ω_* and \tilde{c} , the constraints on \tilde{c} are very different from those in the $\epsilon = -1$ branch case, namely $7.7 < \log_{10} \tilde{c} < 15.9$ (95% C.L.). However, note that the minimal χ^2 for this case is $\chi^2 \approx 688$, that is much higher than that for the $\epsilon = -1$ branch case, which means that the $\epsilon = +1$ branch case is not favored by observations. This can be additionally seen by calculating the corresponding age of the universe, which is much smaller than the Λ CDM value. However, although this branch is not favored by late-times observations, due to that $H^2 \approx \text{const.}$ at early times, it could still play an important role in the inflationary regime.

5 Conclusions

In this work we constrained an alternative 5-dimensional braneworld cosmology using observational data. The difference with the standard braneworld cosmology refers to the adaptation of alternative matching conditions introduced in [16] which generalize the conventional matching conditions. The reasons for this consideration are possible theoretical deficiencies of the standard junction conditions, namely the need for consistency of the various codimension defects and the existence of a meaningful equation of motion at the probe limit. Instead of varying the brane-bulk action with respect to the bulk metric at the brane position and derive the standard matching conditions, we vary with respect to the brane embedding fields in a way that takes into account the gravitational back-reaction of the brane onto the bulk.

The proposed gravitating Nambu-Goto matching conditions may be close to the correct direction of finding realistic matching conditions since they always have the Nambu-Goto probe limit (independently of the gravity theory, the dimensionality of spacetime or codimensionality of the brane), and moreover, with these matching conditions, defects of any codimension seem to be consistent for any (second order) gravity theory. Compared to the conventional 5-dimensional braneworld cosmology, the new one possesses an extra integration constant, which if set to zero reduces the new cosmology to the conventional braneworld one.

In the present work we extended the codimension-1 cosmology of [27] by allowing both a matter and a radiation sector in order to extract observational constraints on the involved model parameters. In particular, we used data from supernovae type Ia (SNIa) and Baryon Acoustic Oscillations (BAO), along with arguments from Big Bang Nucleosynthesis (BBN) in order to construct the corresponding probability contour-plots for the parameters of the theory.

Concerning the first ($\epsilon = -1$) branch of cosmology, we found that the parameters \tilde{V} and \tilde{c} that quantify the deviation from the Randall-Sundrum scenario, are constrained very close to their RS values as expected. However, a departure from Randall-Sundrum scenario is still allowed, and moreover, the corresponding χ^2 is the same for both models. This means that braneworld models with gravitating Nambu-Goto matching condition are in a very good agreement with observations too. The obtained age of the universe is $12.23 \text{ Gyr} \leq t_0 \leq 14.13 \text{ Gyr}$, which is an additional observational advantage of the model. Finally, concerning the fundamental mass scale M , the current age estimations imply that the preferred values of M lie well below the GeV scale.

Concerning the second ($\epsilon = +1$) cosmological branch, which is completely new and with no correspondence in Randall-Sundrum scenario, we extracted the corresponding likelihood contours. Although this case is still compatible with observations, the corresponding minimal χ^2 is much higher than that for the $\epsilon = -1$ branch case, which means that this branch case is not favored by late-times observations. However, although this branch is not favored by late-times observations, due to that $H^2 \approx \text{const.}$ at early times, it could still play an important role in the inflationary regime.

In summary, cosmology with gravitating Nambu-Goto matching conditions offers an extension to the standard Randall-Sundrum scenario. Apart from interesting solutions, we see that it is in agreement with observations since the data allow for a small deviation from Randall-Sundrum cosmology. Therefore, it should be worthy to further study the cosmological implications of the model, such as the inflationary behavior and the late-times asymptotic features.

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A Observational data and constraints

In this Appendix we review the main procedures of observational fittings used in the present work, namely Type Ia Supernovae (SNIa) and Baryon Acoustic Oscillations (BAO).

a. Type Ia Supernovae constraints

We use the Union 2.1 compilation of SNIa data [36] in order to incorporate Supernovae type Ia constraints. This is a heterogeneous data set, which includes data from the Super-

nova Legacy Survey, the Essence survey and the Hubble-Space-Telescope observed distant supernovae.

The χ^2 for this analysis is written as

$$\chi_{SN}^2 = \frac{\sum_{i=1}^N [\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma_{\mu,i}^2}, \quad (\text{A.1})$$

where $N = 580$ is the number of SNIa data points. In the above expression μ_{obs} is the observed distance modulus, which is defined as the difference of the supernova apparent magnitude from its absolute one. Furthermore, $\sigma_{\mu,i}$ are the errors in the observed distance moduli, which are assumed to be uncorrelated and Gaussian, arising from a variety of sources. If we introduce the usual (dimensionless) luminosity distance $D_L(z; a_i)$, calculated by

$$D_L(z; a_i) \equiv (1+z) \int_0^z dz' \frac{H_0}{H(z'; a_i)}, \quad (\text{A.2})$$

with H_0 the present Hubble parameter, then the theoretical distance modulus μ_{th} has a dependence on the model parameters a_i as

$$\mu_{\text{th}}(z) = 42.38 - 5 \log_{10} h + 5 \log_{10} [D_L(z; a_i)]. \quad (\text{A.3})$$

Finally, the marginalization over the present Hubble parameter is performed following [37], which eventually provides the χ^2 likelihood contours for the model parameters that are involved.

b. Baryon Acoustic Oscillation constraints

In order to handle the baryon acoustic oscillation (BAO) observational constraints we use the definition [38]

$$A \equiv D_V(z = 0.35) \frac{\sqrt{\Omega_m H_0^2}}{0.35c} = 0.469 \pm 0.017, \quad (\text{A.4})$$

where c is the light speed. In the above expression we have defined the “volume distance” $D_V(z)$ as

$$D_V(z) \equiv \left[\frac{(1+z)^2 D_A^2(z) z}{H(z)} \right]^{1/3}, \quad (\text{A.5})$$

where

$$D_A \equiv r(z) / (1+z) \quad (\text{A.6})$$

is the angular diameter distance. Finally, the BAO likelihood is written as

$$\chi_{BAO}^2 = \frac{(A - 0.469)^2}{0.017^2}. \quad (\text{A.7})$$

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